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Scattering theory in open quantum systems: Lindblad-type evolutions.

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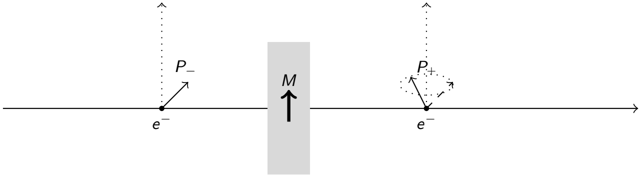
Outline

Physical context.

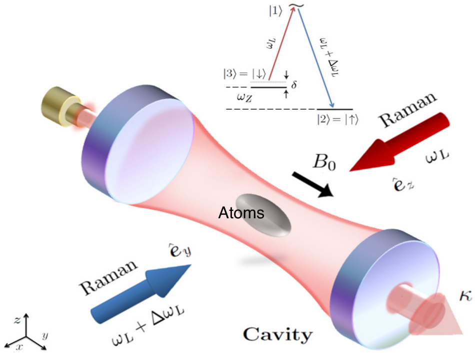
We are interested in analyzing the long-time behavior of Markovian open quantum systems. Such systems appear in various physical contexts, such as solid state physics and quantum optics:

- *Hot spin injection [Weber et al., 2001].* Spin-polarized electrons transmitted through a magnetized film (Fe, Ni).



Incoming e^-: pure state — Outgoing e^-: mixed state

- *Cavity QED.* Laser-driven atoms in a cavity with an electromagnetic field (cavity mode), see e.g. Dimer et al. [2007].



Cavity loss — and eventual spontaneous emission — makes the system (atoms + cavity mode) open.

Modelling: Lindblad-type operators.

These systems can be effectively described by a dynamics $T(\cdot)$ with the following features:

- It is non-unitary, since it results from the reduction of the full (system + environment) dynamics;
- It does not preserve the "purity" of states;
- It is irreversible, i.e. defined only for positive times $t \geq 0$;
- It has the semigroup property $T(t)T(s) = T(s)T(t) = T(t+s)$, for any $t, s \geq 0$;
- It preserves the positivity and trace of states.

Let $\mathscr{H}$ be the Hilbert space associated to the system; $\mathcal{I}_1(\mathscr{H})$ the ideal of trace-class operators. The quantum states form the subset $S_{\|\cdot\|_{\mathcal{I}_1}}(1) \cap \mathcal{I}_1^+$.

Definition (Following Kossakowski [1972]; Lindblad [1976])

A strongly continuous semigroup $\bigl(T(t)\bigr)_{t\geq 0} = \bigl(e^{-it\mathcal{L}}\bigr)_{t\geq 0} \subset \mathcal{B}\bigl(\mathcal{I}_1(\mathscr{H})\bigr)$ is a Lindblad-type evolution if and only if there exist $H_0 = H_0^* \in \mathrm{ClOp}(\mathscr{H})$ and $(C_j)_{j\in J} \subset \mathcal{B}(\mathscr{H})$ such that for any $\varrho \in D(\mathcal{L}) \subset \mathcal{I}_1(\mathscr{H})$:

$$\mathcal{L}\varrho = \underbrace{[H_0, \varrho]}_{\text{“Isolated” dynamics}} \underbrace{-\tfrac{i}{2}\textstyle\sum_{j\in J}\{C_j^* C_j, \varrho\} + i\sum_{j\in J} C_j \varrho C_j^*}_{\text{Interaction with the environment}} .$$

Remarks

- The semigroup $\bigl(T(t)\bigr)_{t\geq 0}$ satisfies all the properties of the previous slide, in particular it maps states into states.
- The C_j are assumed to be bounded to avoid uninteresting technical complications.

Long time asymptotics.

Question 1

Does the Lindblad-type dynamics $\left(e^{-it\mathcal{L}}\right)_{t\geq 0}$ of a given open system behave — for very large times $t \to \infty$ — as the dynamics of an isolated system?

In other words, is it possible to find a unitary dynamics $\left(e^{-it\mathcal{L}_0}\right)_{t\in\mathbb{R}}$ (i.e. $\mathcal{L}_0 = [A, \cdot]$) such that given $\varrho \in \mathcal{I}_1(\mathscr{H})$ there exist a scattering state $\varrho^+ \in \mathcal{I}_1(\mathscr{H})$:

$$\lim_{t\to+\infty} \|e^{-it\mathcal{L}}\varrho - e^{-it\mathcal{L}_0}\varrho^+\|_{\mathcal{I}_1} = 0 \ ?$$

Preliminary assumption

Let $\mathcal{L}$ be a Lindblad-type generator, D_{pp} the space spanned by its eigenvectors. *We suppose that $D = \mathcal{I}_1(\mathscr{H}) \setminus D_{\mathrm{pp}}$ is a closed subspace of $\mathcal{I}_1(\mathscr{H})$.* In addition, we suppose that $\mathcal{L}_0$ has no point spectrum.

- *Wave operators.*

$$\Omega^-(\mathcal{L}_0,\mathcal{L}) = \operatorname*{s-lim}_{t\to+\infty} e^{it\mathcal{L}_0}e^{-it\mathcal{L}}\Big|_D ,$$
$$\Omega^+(\mathcal{L},\mathcal{L}_0) = \operatorname*{s-lim}_{t\to+\infty} e^{-it\mathcal{L}}e^{it\mathcal{L}_0} ;$$

- *Scattering endomorphism.*

$$\sigma = \Omega^-(\mathcal{L}_0,\mathcal{L})\Omega^+(\mathcal{L},\mathcal{L}_0) .$$

1. If Ω^- exists, then the answer to Question 1 is affirmative;
2. If in addition Ω^+ exists and $\mathrm{Ran}(\Omega^+) = D$, then σ exists;
3. If in addition $\mathrm{Ran}(\Omega^-) = \mathcal{I}_1(\mathscr{H})$, then the scattering endomorphism is an isomorphism, and thus invertible.

If (1), (2), and (3) are true, then the theory is *asymptotically complete*.

Assumptions.

We recall that the Lindblad-type generator has the form (for simplicity we set $J = \{0\}$, and $C_0 = C$):

$$\mathcal{L}\varrho = [H_0, \varrho] - \frac{i}{2}\{C^* C, \varrho\} + iC\varrho C^* \ .$$

Assumption 1

There exist a dense subset $\mathscr{E} \subset \mathscr{H}$ such that for any $u \in \mathscr{E}$:

$$\int_{\mathbb{R}} \left\| C^* C e^{-itH_0} u \right\|_{\mathscr{H}} dt < +\infty \ .$$

Assumption 2

There exist a positive constant c_0 (that depends on C and H_0) such that for any $u \in \mathscr{H}$:

$$\int_{\mathbb{R}} \left\| C e^{-itH_0} u \right\|^2_{\mathscr{H}} dt \leq c_0^2 \|u\|^2_{\mathscr{H}} \ .$$

Remarks

- Assumption 2 is that the operator C is H_0-smooth in the sense of Kato [1965/1966].
- We assume decay properties only for operators on the Hilbert space $\mathscr{H}$, and that is sufficient to study Lindblad-type evolutions that act on $\mathcal{I}_1(\mathscr{H})$ — checking assumptions for operators on $\mathscr{H}$ is simpler than for operators on $\mathcal{I}_1(\mathscr{H})$.

We are now in a position to formulate our main results. In the following, $\mathcal{L}_0$ is defined by:

$$\mathcal{L}_0\varrho = [H_0, \varrho] \ .$$

Theorem (F., Faupin, Fröhlich, Schubnel)

- *Suppose that either Assumption 1 or Assumption 2 with $c_0 < 2$ holds. Then*

$$\Omega^+(\mathcal{L}, \mathcal{L}_0) \text{ exists on } \mathcal{I}_1(\mathscr{H}) \; .$$

- *Suppose that Assumption 2 holds with $c_0 < 2$. Then*

$$\Omega^-(\mathcal{L}_0, \mathcal{L}) \text{ exists on } \mathcal{I}_1(\mathscr{H}) \; .$$

- *Suppose that Assumption 2 holds with $c_0 < 2 - \sqrt{2}$. Then the theory is asymptotically complete. More precisely:*

$$\Omega^+(\mathcal{L}, \mathcal{L}_0) \text{ and } \Omega^-(\mathcal{L}_0, \mathcal{L}) \text{ are invertible in } \mathcal{B}\big(\mathcal{I}_1(\mathscr{H})\big) \; ;$$
$$\mathcal{L} = \Omega^-(\mathcal{L}_0, \mathcal{L})^{-1} \, \mathcal{L}_0 \, \Omega^-(\mathcal{L}_0, \mathcal{L}) \; ,$$
$$\mathcal{L}_0 = \Omega^+(\mathcal{L}, \mathcal{L}_0)^{-1} \, \mathcal{L} \, \Omega^+(\mathcal{L}, \mathcal{L}_0) \; .$$

Remarks

- With Assumptions 1 and 2, we can forget about the preliminary assumption on the spectra of $\mathcal{L}$ and $\mathcal{L}_0$: we are able to prove the existence of Ω^+ and Ω^- on the whole $\mathcal{I}_1(\mathscr{H})$.

- Since we considered only bounded C, we can extend the semigroup $(e^{-it\mathcal{L}})_{t\geq 0}$ to a group $(e^{-it\mathcal{L}})_{t\in\mathbb{R}}$, and define

$$\Omega^+(\mathcal{L}_0,\mathcal{L}) = \underset{t\to+\infty}{\text{s-lim}}\, e^{-it\mathcal{L}_0} e^{it\mathcal{L}} \; ;$$
$$\Omega^-(\mathcal{L},\mathcal{L}_0) = \underset{t\to+\infty}{\text{s-lim}}\, e^{it\mathcal{L}} e^{-it\mathcal{L}_0} \; .$$

Under Assumption 2 with $c_0 < 2-\sqrt{2}$ they both exist, and

$$\Omega^-(\mathcal{L}_0,\mathcal{L})^{-1} = \Omega^-(\mathcal{L},\mathcal{L}_0) \quad , \quad \Omega^+(\mathcal{L},\mathcal{L}_0)^{-1} = \Omega^+(\mathcal{L}_0,\mathcal{L}) \; .$$

Sketch of the proof.

- Following Davies [1980a,b], we start studying an auxiliary scattering problem on $\mathscr{H}$:
  - We define the dissipative operator H on $\mathscr{H}$ by

$$H = H_0 - \frac{i}{2} C^* C \ .$$

  - Using our assumptions, we study the wave operators $W_\pm(H, H_0)$, $W_\pm(H_0, H)$, $W_\pm(H^*, H_0)$, and $W_\pm(H_0, H^*)$ of $\mathcal{B}(\mathscr{H})$.
- Using the auxiliary scattering problem, we prove existence of the wave operators $\Omega^+(\mathcal{L}, \mathcal{L}_0)$ and $\Omega^-(\mathcal{L}_0, \mathcal{L})$ by Cook's method.
- Asymptotic completeness is proved again using the properties of the auxiliary scattering problem, and a suitable Dyson series expansion for $\mathcal{L}$.

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